Thickening in the 20th century: a historical perspective

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Abstract

This paper reviews the development of thickening theory and practice during the 20th century. It starts with the early design of the Dorr thickener and describes the progress made in understanding the process in the laboratory and in the plant. The major contributions of Coe and Clevenger, Comings and Kynch are given in detail, and the recent developments by several authors, leading to the phenomenological theory of sedimentation, are presented from the authors' perspective. The known methods of thickener design are mentioned, and the contribution of the authors in this matter is expressed in detail. Finally, some new research and developments, such as high-capacity high-density thickeners and new tools to study and design thickeners, are indicated.

The invention of the Dorr thickener

Clarification and thickening involve the settling of one substance in solid particulate form through a second substance in liquid form. While clarification deals with very dilute suspensions, thickening is performed to produce pulps that are more concentrated. Maybe this is the reason clarification was the first of these operations amenable to mathematical description. The work of Hazen in 1904 was the first analysis of factors affecting the settling of solid particles from dilute suspensions in water. Hazen's analysis shows that detention time is not a factor in the design of settling tanks, but rather that the portion of solid removed is directly proportional to the surface area of the tank, is directly proportional to the settling properties of the solid matter and is inversely proportional to the flow rate through the tank.

The invention of the Dorr thickener in 1905 can be mentioned as the starting point of the modern thickening era. It made the continuous dewatering of dilute pulp possible, whereby a regular discharge of a thick pulp of uniform density took place concurrently with overflow of clarified solution.

Scraper blades or rakes, driven by a suitable mechanism, rotating slowly over the bottom of the tank, which usually slopes gently toward the center, move the material as fast as it settles without enough agitation to interfere with the settling (Dorr, 1915) (Fig. 1).

The first reference on variables affecting sedimentation was made in 1908. Authors such as Nichols (1908), Forbes (1912), Clark (1915), Free (1916) and Ralston (1916) studied the effect of solid and electrolytes concentration, degree of flocculation and temperature in the process.

Mishler (1912, 1918), an engineer and superintendent at the Tigre Mining Co.'s concentrator in the Sonora desert of Mexico, was the first to show by experiment that the rate of settling of slimes is different for dilute and for concentrated suspensions.

While the settling speed of dilute slimes is usually independent of the depth of the settling column, the thick slimes-sedimentation rate increases with the depth of the settling column.

Mishler devised the following formula by means of which laboratory settling test results

Figure 1 — Original Dorr thickener (Dorr, 1936).

Invited feature number 02-326. Original manuscript submitted August 2002. Revised manuscript received and accepted for publication February 2003. Discussion of this peer-reviewed and approved paper is invited and must be submitted to SME Publications Dept. prior to Nov. 30, 2003. Copyright 2003, Society for Mining, Metallurgy, and Exploration, Inc.

Figure 2—Batch settling of a compressible pulp according to Coe and Clevenger (1916).

could be used for continuous thickeners

$$UA = \frac{S}{F} = \frac{D_F - D_D}{\rho_f v_s(D_F)} \quad (1)$$

where

UA is the required area per unit flow rate,

S is the cross-sectional area of the thickener,

F is the feed flow rate,

ρ_f is the density of the fluid,

$v_s(D_F)$ is the velocity of appearance of the supernatant water in a batch settling test with the initial dilution (D_F) of the feed and

D_D is the desired underflow dilution (dilution is the ratio of water and solids in a suspension).

Based on Clark's (1915) results and on their own experiments, Coe and Clevenger (1916) recognized that the settling of an initially homogenous flocculent suspension gives rise to four settling zones. From top to bottom, they distinguished (see Fig. 2) the following zones: A is a clear water zone, B is a zone of constant initial concentration, C is a transition zone of variable concentration and D is a compression zone.

Coe and Clevenger (1916) argued that the *solids handling capacity*, today called *solids flux-density*, has a maximum value in the thickener at a certain dilution (D_k) between the feed and the discharge concentration. Independently from Mishler (1912), they developed the following equation that is similar to Eq (1) but with the feed dilution D_F replaced by the limiting dilution D_k

$$UA = \frac{D_k - D_D}{\rho_f v_s(D_k)} \quad (2)$$

The symbols have the same meaning as in Eq (1). Coe and Clevenger (1916) recommend the determination of the limiting dilution D_k by batch experiment tests. Equation (2), with certain corrections, continues to be the most reliable method of thickener design to date.

Several authors described the settling of suspensions by extending Stokes' equation, or by empirical modeling (Egolf and McCabe, 1937; Ward and Kammermeyer, 1940; Work and Kohler, 1940; Kammermeyer, 1941), but no further important contributions were made on thickening technology until the 1940s. In a review paper, Stewart and Roberts

(1933) give a good idea of the state of the art of thickening in those days:

"The basic theory is old but limitations and modifications are still but partially developed. Especially in the realm of flocculent suspensions is the underlying theory incomplete. Practical testing methods for determining the size of machines to be used are available, but the invention and development of new machines will no doubt be greatly stimulated by further investigation of the many interesting phenomena observed in practice and as fresh problems are uncovered."

Operating variables in a continuous thickener

After Comings published his paper "Thickening of calcium carbonate slurries" (Comings, 1940), at least nine engineering theses were made under his guidance at the University of Illinois in the following decade, mainly on the effect of operating variables in continuous thickening. Comings and co-workers described these findings in an important paper (Comings et al., 1954). They show four zones in a continuous thickener: the clarification zone at the top, the settling zone underneath, the upper compression zone further down and rake action zone at the bottom. Two of the most important features in the operation of a thickener were expressed for the first time. In the first place, that the concentration in the settling zone is nearly constant for a thickener at steady state and that its concentration depends on the rate at which the solid is fed to the thickener and not on the concentration of the feed. It was verified that in most cases the feed was diluted to an unknown concentration on entering the thickener. The second finding was that, for the same feed rate, increasing or decreasing the sediment depth could adjust the underflow concentration.

Roberts (1949) advanced the empirical hypothesis that the rate at which water is eliminated from a pulp in compression is, at all times, proportional to the amount of water left in the sediment

$$D - D_\infty = (D_0 - D_\infty) \exp(-Kt) \quad (3)$$

where

D_0 , D and D_∞ are the dilutions of the pulp at time zero, at time t and at infinite time, respectively.

Equation (3) has sometimes been used for estimating the critical concentration.

Kinematical theory of sedimentation

Coe and Clevenger's (1916) design procedure, the only quantitative knowledge in sedimentation accomplished during the first half of the 20th century, was based on a macroscopic balance of the solid and fluid in a sedimentation vessel and on the observation of the different concentrations established in the thickener. No underlying sedimentation theory existed.

In 1952, G.J. Kynch, a mathematician at the University of Birmingham in Great Britain, presented in his celebrated paper "A theory of sedimentation" a kinematical theory of sedimentation based on the propagation of concentration waves in the suspension (Kynch, 1952). The suspension is considered a continuum, called ideal suspension, and the sedimentation process is represented by the continuity equation of the solid phase

$$\frac{\partial \phi}{\partial t} + \frac{\partial f_{bk}(\phi)}{\partial z} = 0, \quad 0 \leq z \leq L, \quad t > 0 \quad (4)$$

where

ϕ is the volume fraction of solids;
 $f_{bk}(\phi) = \phi v_s$ is the *solid flux-density function*, satisfying the restrictions
 $f_{bk}(0) = f_{bk}(\phi_{\max}) = 0$ and $f_{bk}(\phi) < 0$
for $0 < \phi < \phi_{\max}$; and
 v_s is the solid phase velocity.

Equation (4) is considered with suitable initial and boundary conditions.

In solutions of scalar conservation laws such as Eq. (4), discontinuities appear in the concentration as a function of height and time. At such discontinuities, Eq. (4) is no longer valid and is replaced by the jump condition $\sigma = [f_{bk}]/[\phi]$, where σ is the speed of propagation of the discontinuity and $[\cdot]$ represents the jump in a property from one side of the discontinuity to the other.

The basic assumption of Kynch's theory is that, at any point in the flow field, the settling velocity is a function of the local solids concentration ϕ only. Kynch shows that, if the form of the flux-density function and the initial concentration $\phi(z, 0)$ of a suspension are known, then the solution of Eq. (4) can be constructed by the method of characteristics and this procedure describes the complete sedimentation process.

This paper had the greatest influence in the development of thickening thereafter. When Comings moved from Illinois to Purdue University, research on thickening continued there for another ten years. Although Comings soon moved on to Delaware, work continued at Purdue under the direction of Paul Shannon. A Ph.D. thesis by Tory (1961) and M.S. theses by Stroupe (1962) and De Haas (1963) analyzed Kynch's theory. Their results were published in a series of joint papers (Shannon et al., 1963, 1964; Shannon and Tory, 1965, 1966). The proof of the validity of Kynch's theory by experiments with glass beads was published in the paper of Shannon et al. (1964). Batch sedimentation was regarded as the process of propagation of concentration waves upwards from the bottom of the settling vessel.

The concept of an *ideal thickener* was presented for the first time by Shannon and Tory (1966) and complemented by Bustos et al. (1990, 1999) and by Concha and Bustos (1992). The ideal thickener is a vessel with no friction at the walls. For batch sedimentation, the ideal thickener is called *settling column* and for continuous sedimentation, it receives the name of *ideal continuous thickener (ICT)* (see Fig. 3).

The ICT has a surface source as feed and a surface sink as an underflow

$$\begin{aligned} f_F(t) &= Q_F(t)\phi_F(t)/S \\ \text{and} \\ f_D(t) &= Q_D(t)\phi_D(t)/S \end{aligned} \quad (5)$$

where

Q_F and Q_D are the volume flow rate of pulp ϕ_F and ϕ_D and are the concentration at $z = L$ and $z = 0$, respectively, and S is the cross-sectional area of the equipment.

If $q(t)$ is defined as the volume flow rate per unit area of the sedimentation vessel at the underflow, then Kynch's equation for continuous sedimentation is

Figure 3 — Concept of an ideal thickener.

$$\frac{\partial \phi}{\partial t} + \frac{\partial}{\partial z} (q\phi + f_{bk}(\phi)) = 0 \quad (6)$$

Kynch's theory was so successful that there existed a tendency to extend its validity in still another way. Several authors, among them Fitch (1983) and Font (1988), tried to modify Kynch's theory to account for the compressive effects. This approach encountered several problems that could not be solved within the theory. A different theory was needed.

At the beginning of the 1980s, mathematicians from the University of Concepción in Chile and the Technical University Darmstadt, and later from the University of Stuttgart in Germany, in collaboration with the Department of Metallurgical Engineering at the University of Concepción, became interested in the field of conservation laws. They started a comprehensive study of the mathematical aspects of sedimentation. Their results were published in the doctoral theses by Bustos (1984), Kunik (1989) and Bürger (1996), in several papers by Bustos, Bürger, Concha, Wendland and other collaborators, in the book by Bustos et al. (1999) and the habilitation thesis by Bürger (2002). The first step of their work was to establish a rigorous framework for the theory of ideal suspensions (Kynch theory). Using the method of characteristics, Bustos and Concha (1988) and Concha and Bustos (1991) constructed entropy weak solutions of Kynch's problem, in which zones of constant concentrations are separated by shocks, rarefaction waves or combinations of these. For flux density functions with two inflexion points, they presented five modes of sedimentation. These findings were complemented with two other modes of sedimentation in Bustos et al (1999) and Bürger and Tory (2000) (see Figs. 4 and 5).

Similar solutions can be constructed by the method of characteristics for continuous sedimentation of ideal suspensions (Bustos et al., 1990; Bustos and Concha, 1996; Concha and Bustos, 1992; Concha and Bürger, 1998). The solution of Eq. (6) leads to three *modes of continuous sedimentation* (MCS). For a detailed and a concise overview of the construction of weak solutions, see Bustos et al. (1999) and Concha and Bürger (1998), respectively.

Based on the construction of weak solutions, Bustos et al. (1990b) formulated a simple control model for continuous

Figure 4 — First three possible modes of sedimentation for a flux density function with two inflexion points (after Bürger and Tory, 2000).

sedimentation of ideal suspensions in ICT. It is shown that certain steady states can always be recovered after perturbation of the feed flux density by solving two initial and boundary value problems at known times and with parameters q and ϕ_L that can be calculated a priori.

The effect of vessels with varying cross sections and boundary conditions were studied by Diehl (1997, 1999) and by Bürger et al. (2003a, 2003b, 2003c, 2003d). Diehl shows that, at the bottom, the suspension concentration increases due to the conical shape. This increase in concentration was previously shown by Shannon and Tory (1966). Bürger et al. (2003a, 2003c) indicate that due to the varying cross section, characteristics and iso-concentration lines do not coincide and that to obtain results numerical methods have to be used. Chancelier et al. (1994), Diehl (1997) and Bürger et al. (2002, 2003a, 2003b, 2003c) treat the feed, the discharge and the overflow mechanism as discontinuities of the flux density function adding a source term at the feed level, thus making boundary conditions unnecessary.

Dynamical theory of sedimentation

Experience by several authors, among them Yoshioka et al. (1957), Hassett (1958, 1964, 1968), Shannon et al. (1963), and

Scott (1968a, 1968b), demonstrated that, while Kynch's theory accurately predicts the sedimentation behavior of a suspension of equally-sized small rigid particles, this is not the case for compressible suspensions.

Although Behn (1957) was the first writer who attempted to apply consolidation theory for the settling of compressible slurries, Mompei Shirato and co-workers (Shirato et al., 1970) were the first who indeed solved the combined settling-consolidation problem. Using material coordinates, they obtained settling curves and excess pore pressure profiles. It took another five years for Adorján (1975, 1976) to present his ad-hoc theory of sediment compression, giving the first satisfactory method of thickener design.

At about the same time, a group of research workers in Brazil gave the phenomenological sedimentation theory a proper framework. A strong and important research on thickening, and in general in the field of flow through porous media, was going on in Brazil in the 1970s. At the Graduate School of the Federal University of Rio de Janeiro, COPPE, several researchers and graduate students, among them Giulio Massarani, Affonso Silva Telles, Rubens Sampaio, I-Shih Liu, José Teixeira Freire and João D'Avila (D'Avila, 1976, 1978; D'Avila and Sampaio, 1977; D'Avila et al., 1978), to

Figure 5 — Four last possible Modes of Sedimentation for a flux density function with two inflexion points (after Bürger and Tory, 2000).

mention only a few, were involved in the application of the newly developed mathematical tool, the *Theory of Mixtures* of continuum mechanics, to particulate systems. Their findings were presented at the Porous Media Symposia organized every year since 1973 by Massarani and his group. This series of events received in 1996 the new name of the Brazilian Congress on Particulate Systems. Contributions presented at these meetings are well documented in yearly-published annals.

With strong ties with the Brazilian researchers, the first author of this review worked in the same direction at the University of Concepción in Chile. Initial findings were

presented in the Engineering Theses by Bascur (1976) and Barrientos (1978) and at the Engineering Foundation Conference on Particle Technology in New Hampshire, USA, in 1980.

Independently, Kos (1977) used the Theory of Mixtures to set up a boundary value problem for batch and continuous sedimentation. Thacker and Lavelle (1977) used the same theory for incompressible suspensions. At the end of the 1970s and during the 1980s, several papers, for example, Buscall and White (1987), Auzerais et al. (1988), Landman et al. (1988), Bascur (1989) and Davis and Russel (1989), show that the phenomenological model, based on the Theory of

Figure 6 — Solid Kynch flux density for a copper tailing (Becker, 1982).

Figure 7 — Solid effective stress for a copper tailing (Becker, 1982).

Mixtures, was well accepted by the international scientific community.

The phenomenological theory of sedimentation is based on the Theory of Mixtures. It assumes that a particulate system is composed of two superimposed continuous media obeying the following restriction:

- the solid particles are small with respect to the containing vessel and have the same density, size and shape;
- the solid and the fluid are incompressible;
- there is no mass transfer between components; and
- gravity is the only body force.

For an ICT, the local mass and momentum balances, together with constitutive equations for the stresses and forces, give rise to the following nonlinear degenerate parabolic differential equation (Bürger and Concha, 1998, Bürger et al.,

1999, Bustos et al., 1999) describing the sedimentation of flocculated suspensions

$$\frac{\partial \varphi}{\partial t} + \frac{\partial}{\partial z} (q\varphi + f_{bk}(\varphi)) = \frac{\partial}{\partial z} \left(- \frac{f_{bk}(\varphi) \sigma_e(\varphi)}{\Delta \rho \varphi g} \frac{\partial \varphi}{\partial z} \right) \quad (7)$$

where

q is a constant equal to the volume average velocity, $f_{bk}(\varphi)$ is the batch Kynch flux density function and $\sigma_e(\varphi)$ is the solid effective stress.

To complete the dynamic process, constitutive equations must be postulated for the functions $f_{bk}=f_{bk}(\varphi)$ and $\sigma_e=\sigma_e(\varphi)$, for example (Michaels and Bolger 1962, Tiller and Leu 1980)

$$f_{bk} = u_{\infty} \varphi (1 - \varphi / \varphi_{\max})^c,$$

$$\sigma_e = \begin{cases} 0 & \text{for } \varphi \leq \varphi_c \\ \sigma_0 \left\{ (\varphi / \varphi_c)^n - 1 \right\} & \text{for } \varphi > \varphi_c \end{cases} \quad (8)$$

with $n > 1$ and $\sigma_0 > 0$

where

u_{∞} is the settling velocity of a single floc in an infinite medium,
 $\varphi_{\max}$ is the maximum concentration attainable and
 c is a number > 1 .

For values of $\varphi \leq \varphi_c$, Eq. (7) reduces to the first-order hyperbolic equation (continuous Kynch equation)

$$\frac{\partial \varphi}{\partial t} + \frac{\partial}{\partial z} (q\varphi + f_{bk}(\varphi)) = 0 \quad \text{for all } 0 \leq \varphi \leq \varphi_c \quad (9)$$

Equation (9) shows that the Kynch theory of ideal suspensions is valid for all types of suspensions, compressible or not, for regions where the concentration is less than the critical (Kynch regime). Figures 6 and 7 show these functions for the case of copper tailings.

Batch sedimentation. Bürger et al. (2000a) and Bürger and Karlsen (2001) devised numerical methods for solving Eq. (7). These numerical methods have a built-in property to reproduce appropriately discontinuities of the entropy solutions, especially the *suspension-sediment interface*, where the equation changes type, without the necessity to track them explicitly; that is, the scheme has the so-called shock-capturing property. Bürger et al. (2000c) and Garrido et al. (2000) show the application of this method to several batch sedimentation processes published in the literature. Figure 8 gives the result for one of those cases.

Figure 9 shows a solution for the continuous case. After operating the thickener with a given feed rate f_F^1 and underflow rate q^1 , the underflow is abruptly changed to q^2 and later to q^3 . After a while, the feed rate is adjusted to reach steady states. In Fig. 9, (b) shows the change in the sediment level, (d) shows the change in underflow concentration and (a) shows the change in conjugate concentration.

Steady state. Considering Eq. (7) at steady state, one can obtain the following equations for the steady state of a continuous thickener

$$f = q\phi + f_{bk}(\phi) \left(1 + \frac{\sigma_e(\phi)}{\Delta\rho\phi g} \frac{\partial\phi}{\partial z} \right) = f_F \equiv \text{constant}$$

$$q = \frac{f_F}{\phi_D} \equiv \text{constant} \quad (10)$$

For steady state, both the solid flux density and the volume average velocity are constant along the thickener and are determined externally.

On entering the thickener the suspension is diluted to $\phi(L) \equiv \phi_L < \phi_c$ and, because Kynch's equation is valid,

$$f_F = f(\phi(L)) \equiv q\phi_L + f_{bk}(\phi_L) \quad (11)$$

The conjugate concentration ϕ_L may be calculated from Eq. (11) or graphically from Fig. 10. The concentration gradient may be calculated by integration of Eq. (10) from $z(\phi_D) = 0$ to $z_c = z(\phi_c)$ (see Fig. 11).

Thickener design

Since the first thickener design method by Coe and Clevenger (1916), many other methods have been developed, most of them based on Kynch's theory of sedimentation. Talmage and Fitch (1955) devised such a method. They affirmed that one settling plot had all the information needed to design a thickener. Because in the settling plot, the water-suspension interface slope gives the settling velocity of the suspension, the slope at different times should represent the settling velocity at different concentrations. They used this information in conjunction with Coe and Clevenger's method to devise their method. With a different view, Yoshioka et al. (1957) and Hassett (1958, 1964a) used the solid flux-density function to interpret the operation of a continuous thickener and devise a method for thickener design. Although Kynch's theory cannot be regarded as appropriate for flocculent suspensions (see next section), thickener manufacturers still use and recommend Talmage and Fitch's method of thickener design. In 1975 Adorján (1975, 1976) presented his ad-hoc theory of sediment compression, giving the first satisfactory method of thickener design. For details on design procedures (see Concha and Barrientos, 1993; Bustos et al., 1999).

Bustos et al. (1999) show that the unit area, that is, the thickener area needed to treat a certain daily flow rate of dry solids, is given by

$$UA \geq \max_{\phi_L \leq \phi \leq \phi_D} G(\phi, \phi_D) \quad (12)$$

where

$$G(\phi, \phi_D) := \frac{1}{\rho_s f_{bk}(\phi)} \left(\frac{\phi}{\phi_D} - 1 \right), \quad (13)$$

for $\phi_L \leq \phi \leq \phi_D$

Figure 12 shows the function $G(\phi, \phi_D)$ and the value of UA for the case of copper flotation tailings (Garrido et al., 2003b).

In designing a thickener, the desired mass feed flow rate $F = \rho_s f_F / S$ and underflow concen-

Figure 8 — Solution of the batch sedimentation equation for flotation tailings (Holdich and Butt, 1997).

Figure 9 — Dynamic response of the sediment surface, the conjugate concentration, lines of equal concentration and the discharge concentration for the continuous sedimentation of a flocculated suspension in the transient state for step changes of $q(t)$ and $f_D(t)$ (Bürger et al., 2000c).

Figure 10 — Flux density function vs. concentration for a continuous thickener.

Figure 11 — Concentration profile in a continuous thickener.

Figure 12 — Function $G(\phi, \phi_D)$ showing the value of UA (Garrido et al., 2003b).

tration (ϕ_D), together with the thickening parameters f_{bk} and the solid and liquid densities ρ_s, ρ_l must be known. With these values, the function $G(\phi, \phi_D)$ can be calculated for any value of ϕ , as shown in Fig. 10 (see Garrido et al., 2003b).

The problem in calculating the maximum $G(\phi, \phi_D)$ is that the conjugate concentration ϕ_L is unknown until the value of f_F is known, and to calculate this term ($f_F = -F/\rho_s S$) one needs S , which is the result one is seeking. Therefore, the problem is undetermined, and, to solve it, a value of ϕ_L must be assumed (for example: $\phi_L \approx 0.01$) and the UA calculated in the range of ϕ_L to ϕ_D . With this choice of UA , a value of S is calculated leading to the calculation of f_F and q . With all these values, the concentration profile in the thickener is calculated using Eq. 10, and the sediment height z_c is obtained.

The next steps of the design are the choice of a value for z_c (for example, 20% of the height L of the thickener) and iteration, changing the value of S , to obtain a corrected unit area (UA). Finally, the thickener diameter obtained is approximated to the next integer and the simulation is repeated to obtain the conjugate concentration, the thickener area and the concentration profile.

Topics of current research and some open problems

Most of the theoretical and practical knowledge that was developed during the 20th century in the field of sedimentation-consolidation applies to the so-called conventional thickener. These thickeners have the distinctive feature that the ratio of diameter to height is large and that the feed pulp dilutes on entering the equipment. Conventional thickeners may have diameters of more than 100 m (330 ft). Two other types of thickeners exist, the high-capacity and the high-density thickeners. The first one is characterized by a feeding location in the sediment, by which the pulp increases its concentration on entering the equipment and the settling zone is avoided. The high-density thickener has a greater height to diameter ratio, with heights reaching to 20 m (66 ft), which permits a highly concentrated underflow.

High-capacity thickeners may be modeled in a similar way as conventional thickeners. In 1995, the first author of this review published a first attempt at modeling these types of thickeners (Concha et al., 1995), but it was only recently that more research has been done on this subject. Chancelier et al. (1994), Diehl (1997) and Bürger et al. (2002) treat the feed, the discharge and the overflow mechanism in a thickener as discontinuities of the flux density functions, adding a source term at the feed level, thus making it possible to treat conventional and high-capacity thickeners with the same equations. So far, these works have been done for ideal suspensions; the application to flocculent suspensions is still lacking.

Although the first high-density thickener was put in operation in 1973 (Chandler, 1983), the mechanism of sedimentation-consolidation in this equipment has not been studied in detail. Chandler (1983) expresses the view that, in this equipment, consolidation of the sediment is due to channeling through the compacted bed. Research is needed to verify if this type of mechanism produces a constitutive equation that is in accord with experimental evidence. These studies are important due to the tendency of mining companies today of disposing the tailings as pastes (Jewell, 2002; Rice, 2002; Slottee, 2002; Chambers et al., 2002).

New tools have recently been used to analyze sedimentation, consolidation and thickener design. Computational Fluid Dynamics (CFD) was used to analyze the dilution and the flocculant dispersion in feedwells and to determine the forces

of sediment on rakes (Köck and Concha, 1999; Farrow et al., 2000). Quispe et al. (2000, 2001) used discrete models for simulating the sedimentation of ideal suspensions and were able to predict their permeability.

The authors have been collaborating in the formulation, analysis and numerical solution of mathematical models for solid-liquid separation processes for more than eight years. This cooperation has substantially been supported by the German Research Foundation (DFG) through the Collaborative Research Center (SFB) 404 *Multifield Problems in Continuum Mechanics* at the University of Stuttgart and by the Chilean Research Council CONICYT and Fundación Andes through Fondef and various grants.

The preceding review has given a flavor of the benefits of the authors' mathematical approach to sedimentation and thickening, which follows Truesdell's (1966) statement that the characteristics of a good theory is that the physical concepts themselves are made mathematical at the outset and that mathematics should be used to formulate the theory and to obtain solutions. At this point, it should be clearly stated that strongly degenerate parabolic equations, such as Eq. (7) are by no means standard objects in mathematics. In fact, the application to solid-liquid separation processes of flocculated suspensions, sketched here for the case of gravity thickening, has given rise to intense and genuinely new mathematical research. The essential difficulty arises from the fact that, due to the nonlinear nature of the problem, it is not possible to predict beforehand where the sediment level corresponding to the type-change interface between the hindered settling and the compression regions, marked by the isoline of the critical concentration ϕ_c is located.

For some subtle reasons, associated with the discontinuity of the function $\sigma_e(\phi)$, it is not even possible to state an analogue of the jump condition, such as that stated above, for these equations. Nevertheless, the mathematical analysis by Bürger et al. (2000), as well as the numerical analysis by Evje and Karlsen (2000) (see also Bürger and Karlsen, 2001), clearly shows that the continuous sedimentation model defined by Eq. (7) and appropriate initial and boundary conditions has a unique solution, and that reasonably simple numerical schemes probably converge to such solutions. This means that they can be implemented into reliable user-friendly simulation tools (Garrido et al., 2003a, 2003b). The authors attribute the decisive superiority of this approach to the fact that, due to the degeneracy, one scalar equation describes both the hindered settling and the compression zone of the sediment and that there is no need to track the type-change interface. This is clearly opposed to other models (Stamatikis and Tien, 1992; Diplas and Papanicolaou, 1997), where the necessity to introduce moving boundary conditions drastically complicates the treatment.

The authors have invested a great deal of effort into extending the continuous-sedimentation model to other solid-liquid separation operations such as centrifugation (Bürger and Concha, 2001) and pressure filtration of flocculated suspensions (Bürger et al., 2001).

The discussion of this review has been limited to models of continuous sedimentation in which there is only a thickening zone, that is, the feed inlet and the outlet are identified with boundary conditions. Such models are easily amenable to mathematical analysis and numerical simulation, as seen in Fig. 8. However, a clarification zone located above the feed level is still lacking. The latest models for continuous sedimentation are so-called clarifier-thickener units, in which a submerged feed source is modeled as a singular source term,

which gives rise to discontinuous changes of the flux density function. So far, it has been possible to provide sound mathematical and numerical methods for such units for ideal (non-flocculated) suspensions, see Bürger et al. (2002, 2003a, 2003b, 2003c).

Nomenclature

c	parameter in Michaels and Bolger's Eq. (8)
$f_{bk}(\phi)$	batch solid flux density function (Kynch density function)
f_F	feed solid flux density
f_D	underflow solid flux density
g	acceleration of gravity
n	parameter in constitutive equation for the solid effective stress, Eq. (8)
q	volume flow rate per unit cross sectional area (volume average velocity)
t	time
u_∞	settling velocity of a particle (floc) in an infinite medium
v_f	fluid component velocity
v_s	solid component velocity
v_r	relative solid-fluid velocity
z	upwards-pointing vertical space coordinate
z_c	vertical spatial coordinate at the sediment-suspension (water) interface ($\phi = \phi_c$)
D	dilution at time t
D_D	underflow dilution; weight of water per weight of dry solid
D_F	feed dilution
D_k	limiting dilution
D_0	dilution at time 0
D_∞	dilution at infinite time
F	solid feed flow rate
$G(\phi, \phi_D)$	unit area function defined by Eq. (13)
K	first order rate constant
L	Initial height of the sedimenting slurry
$MS - i$	modes of sedimentation with $i = 1, 2, \dots, 7$
Q_F	feed pulp volume flow rate
Q_D	underflow pulp volume flow rate
S	thickener cross sectional area
UA	unit area; area per unit flow rate
ρ_s	solid material density
ρ_f	fluid material density
$\Delta\rho$	$\rho_s - \rho_f$
ϕ	volume fraction of solids
ϕ_c	critical concentration
ϕ_L	conjugate concentration (concentration at $z = L$)
$\phi_{\max}$	maximum value of the solid volume fraction
σ	displacement velocity of a discontinuity
σ_e	solid effective stress
$\sigma_e(\phi)$	$d\sigma_e/d\phi$
σ_0	parameter in constitutive equation for the solid effective stress, Eq. (8)

Acknowledgments

The preparation of this work was made possible through Fondef project D00-T-2027 and Fundación Andes project C-13634 at the University of Concepción, Chile, and through the Sonderforschungsbereich 404 at the University of Stuttgart, Germany.

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