Software for the design and simulation of gravity thickeners

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Abstract

Based on the steady state and the transient solutions of the phenomenological theory of sedimentation, a software was developed for the design and simulation of batch and continuous thickening. The paper presents a brief description of the phenomenological theory of sedimentation and its application to batch and continuous thickening with two appropriate initial and boundary conditions. The software permits the simulation of batch sedimentation and the design and simulation of continuous thickeners. For batch thickening, the initial and critical concentration and the height of the initial suspension have to be entered together with the parameters of the flux density function and the effective solid stress. The output is a settling plot showing as many lines of constant concentration as requested and a plot of the concentration profile for several times. For continuous thickening, the program calculates the unit area and the area of a thickener required to treat a given feed rate. The input is the solid feed rate and the required underflow concentration, together with the thickening parameters. One example shows the application.

Keywords: Tailings; Dewatering; Thickening; Modeling; Simulation

1. Introduction

The phenomenological theory of sedimentation of flocculated suspensions, which was developed during the seventies, is well accepted today by research workers worldwide (Landman and White, 1994; Concha et al., 1996; Bustos et al., 1999; Bürger et al., 2000a, b; Garrido et al., 2000). Bürger et al. (2000c) (see also Bürger and Karlsen, 2001) presented an efficient numerical solution to the final sedimentation equation, based on a generalization of the Engquist-Osher method, which permits the simulation of batch settling curves and of transient continuous thickening on a personal computer.

In a previous work, (Garrido et al., 2003) we used this numerical scheme to develop a simulator for batch and continuous thickening. The objective of this work is to develop software capable of designing a continuous thickener and predicting its capacity, its concentration profile and its solid inventory.

2. Mathematical model

A suspension is said to be in hindered settling when the particles fall independently one from another and it is in consolidation when the particles touch each other permanently during the fall. The concentration at which the process changes from settling to consolidation (or compression) is the critical concentration $\varphi_c$. 

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Assume that all particles are small with respect to the container and have the same density. Furthermore, assume that the solid and the fluid components are incompressible; that there is no mass transfer between components, that the suspension is completely flocculated before the sedimentation starts and that all flocs settle at the same terminal velocity. Under such conditions, sedimentation is characterized by the volume fraction of solid \( \phi(z, t) \), by the Kynch solid flux density function \( f_{bk}(\phi) \), by the volume average velocity \( q(t) \) and by the solid effective stress \( \sigma_e(\phi) \). These four variables must obey the field equation (1), the constitutive equations (2) and (3), and the prescribed condition (4):

\[
\frac{\partial \phi}{\partial t} + \frac{\partial}{\partial z}(q(t)\phi + f_{bk}(\phi)) = -\frac{\partial}{\partial z}\left( f_{bk}(\phi) \frac{\sigma'_e(\phi)}{\Delta \rho \phi g} \frac{\partial \phi}{\partial z} \right),
\]

\[0 \leq z \leq L, \quad t > 0,
\]

\[f_{bk}(\phi) = u_c \phi(1 - \phi/\phi_{max}), \quad \phi \leq \phi_c,
\]

\[\sigma_e(\phi) = \begin{cases} 
0 & \text{for } \phi \leq \phi_c, \\
\exp(\beta \phi) \text{or } \sigma_0 \left( \frac{\phi}{\phi_c} \right)^n - 1 & \text{for } \phi > \phi_c,
\end{cases}
\]

\[q(t) \leq 0.
\]

The special characteristics of each material is described by the constitutive functions \( f_{bk}(\phi) \) and \( \sigma_e(\phi) \), where \( f_{bk}(\phi) \leq 0, f_{bk}(0) = f_{bk}(1) = 0 \), for \( 0 \leq \phi \leq 1 \). Here \( x, \beta, \sigma_0, n, \phi_{max} \) and \( c \) are positive numbers, while \( u_c < 0 \). Typical constitutive functions are shown in Figs. 1 and 2. We see that Eq. (1) is of first-order hyperbolic type for \( \phi \leq \phi_c \) or \( \phi > 1 \) and of second-order parabolic type for \( \phi_c < \phi < 1 \). Several papers have discussed appropriate functions \( f_{bk}(\phi) \) and \( \sigma_e(\phi) \) for a variety of materials (Bürger et al., 2000a; Garrido et al., 2000, 2003).

For the case of batch sedimentation Eq. (1) becomes:

\[
\frac{\partial \phi}{\partial t} + \frac{\partial}{\partial z}(q(t)\phi + f_{bk}(\phi)) = -\frac{\partial}{\partial z}\left( f_{bk}(\phi) \frac{\sigma'_e(\phi)}{\Delta \rho \phi g} \frac{\partial \phi}{\partial z} \right) = 0
\]

with boundary conditions:

\[\phi(z, 0) = \phi_0 \quad \text{for } 0 \leq z \leq L,
\]

\[f_{bk}(\phi) \left( 1 + \frac{\sigma'_e(\phi)}{\Delta \rho \phi g} \frac{\partial \phi}{\partial z} \right) = 0 \quad \text{for } t > 0.
\]

2.1. Batch sedimentation

For continuous thickening, Eq. (1) is valid and we prescribe a volumetric average velocity \( q(t) < 0 \) as a control function. Then, the boundary conditions for the feed and discharge fluxes are (Bürger et al., 1999):

\[q(t)\phi + f_{bk}(\phi) \left( 1 + \frac{\sigma'_e(\phi)}{\Delta \rho \phi g} \frac{\partial \phi}{\partial z} \right) \bigg|_{z=L} = f_F(t), \quad t > 0,
\]

\[q(t)\phi + f_{bk}(\phi) \left( 1 + \frac{\sigma'_e(\phi)}{\Delta \rho \phi g} \frac{\partial \phi}{\partial z} \right) \bigg|_{z=0} = f_D(t), \quad t > 0,
\]

where \( f_F \) and \( f_D \) are the solid flux density at the feed and discharge, given respectively by (Concha et al., 1996):

\[f_F(t) = -\frac{Q_F(t)\phi_F(t)}{S} = -\frac{F(t)}{\rho_S S},
\]

\[f_D(t) = -\frac{Q_D(t)\phi(0, t)}{S} = q(t)\phi_D(t),
\]

where \( Q_F(t) \geq 0, F(t) \geq 0 \), and \( \phi_F(t) \) and \( \phi_D(t) \) are the volume and mass flowrate and the solid volume fraction fed to the thickener, \( \rho_s \) is the solid density and \( S \) is the cross-sectional area of the thickener, \( Q_D(t) \geq 0 \), and \( \phi_D = \phi(0, t) \)
are the volume flowrate and the solid concentration of pulp at the underflow of the thickener respectively. Note that \( f_1(t) \) may be prescribed, while \( f_D(t) \) is part of the solution of the transient problem, because \( \phi_D = \varphi(0, t) \) is unknown beforehand.

Since in the settling region of the thickener the concentration is less than the critical, \( \sigma'_c(\varphi) = 0 \) for \( \varphi < \varphi_c \), the concentration \( \varphi_L \) at \( z = L \) may be obtained by seeking the smallest solution of (Bustos et al., 1999):

\[
    f_F(t) = q(t)\varphi_L(t) + f_{bk}(\varphi_L(t)).
\]

### 2.3. Continuous thickening at steady state

Bustos et al. (1999) established that the necessary condition for a steady state to be admissible is that the following inequality is valid:

\[
    q\varphi + f_{bk}(\varphi) \leq f_F, \quad \text{for } \varphi_L \leq \varphi \leq \varphi_D. \tag{13}
\]

One of the consequences of Eq. (13) is that the concentration in the compression zone, where \( \varphi \) varies between \( \varphi_L \) and \( \varphi_D \), is increasing with depth. To see this, we use the ordinary differential equation obtained by integrating the time-independent version of Eq. (1) with respect to \( z \):

\[
    \frac{dz}{d\varphi} = -\frac{\Delta\rho \varphi g}{\sigma'_c(\varphi)f_{bk}(\varphi)}(q(\varphi - \varphi_D) + f_{bk}(\varphi)) \quad \text{for } z > 0. \tag{14}
\]

Now observe that, in view of (14), the right-hand side of (13) is always negative for \( \varphi_L \leq \varphi \leq \varphi_D \), such that \( d\varphi/dz < 0 \) in the compression zone.

Inequality (13) expresses the fact that, in a flux density function versus concentration plot, the flux density function should be below the horizontal straight line representing \( f(\varphi) = f_F \). Fig. 3 shows three valid steady states for the same underflow rate.

For these steady states, the concentration profile in the thickener can be calculated by integrating Eq. (14) from \( z = 0 \) with \( \varphi(0) = 0 \) to \( z = z_c \) with \( \varphi(z_c) = \varphi_c \). For a given volume average velocity \( q \), the choice of \( \varphi_D \) and the existence of a steady state concentration profile are subject to admissibility conditions, which are discussed in detail Bürger et al. (1999).

Fig. 4 shows the concentration profile for the cases of Fig. 3.

### 3. Thickener design

Since at steady state \( q = f_F/\varphi_D \), substituting this term into (13) yields

\[
    f_F \geq \frac{f_F}{\varphi_D} \varphi + f_{bk}(\varphi) \quad \text{for } \varphi_L \leq \varphi \leq \varphi_D. \tag{15}
\]

Dividing by \( \varphi \) and rearranging we obtain

\[
    f_F \geq \frac{f_{bk}(\varphi)}{(1 - \frac{\varphi}{\varphi_D})} \quad \text{for } \varphi_L \leq \varphi \leq \varphi_D.
\]

By definition, (see (10)), \( F/S = -\rho_s f_F \), therefore

\[
    F \leq \frac{\rho_s f_{bk}(\varphi)}{\left(\frac{\varphi}{\varphi_D} - 1\right)} \quad \text{for } \varphi_L \leq \varphi \leq \varphi_D. \tag{15}
\]

Introducing the function

\[
    G(\varphi, \varphi_D) := \frac{1}{\rho_s f_{bk}(\varphi)} \left(\frac{\varphi}{\varphi_D} - 1\right) \quad \text{for } \varphi_L \leq \varphi \leq \varphi_D.
\]

we can write

\[
    S \geq G(\varphi, \varphi_D) \quad \text{for } \varphi_L \leq \varphi \leq \varphi_D.
\]
If the unit area is defined as
\[ UA := \max_{\varphi_L \leq \varphi \leq \varphi_D} S/F(\varphi, \varphi_D) \]  (17)
then,
\[ UA \geq \max_{\varphi_L \leq \varphi \leq \varphi_D} G(\varphi, \varphi_D). \]  (18)

Fig. 5 shows the function \( G(\varphi, \varphi_D) \) for a given case.

In designing a thickener, the desired feed flow rate \( F \) and underflow concentration \( \varphi_D \), together with the thickening parameters \( f_{bk}(\varphi) \), \( \varphi_c \) and \( \sigma_c(\varphi) \) and the solid and liquid densities \( \rho_s, \rho_l \) must be known. With these values, the function \( G(\varphi, \varphi_D) \) can be calculated for any value of \( \varphi \) (Fig. 6).

### 4. Numerical solution and batch sedimentation

We have already said that the degenerate parabolic balance equation representing the sedimentation-consolidation reduces to a hyperbolic equation for the hindered settling region, where concentrations are less or equal to the critical, and to a parabolic equation for the consolidation region with concentrations greater than the critical. The location of the interface between these two regions forms part of the solution.

It is well known that solutions to the hyperbolic equation are discontinuous in general and that a discontinuity between two concentrations \( \varphi^+ \) and \( \varphi^- \) travels at the speed \( \sigma(\varphi^+, \varphi^-) \) given by the Rankine-Hugoniot condition:
\[ \sigma(\varphi^+, \varphi^-) = \frac{f_{bk}(\varphi^+)}{\varphi^+} - \frac{f_{bk}(\varphi^-)}{\varphi^-}, \quad \varphi^+, \varphi^- < \varphi_c. \]  (19)

In particular, the settling rate of an initially homogeneous suspension of concentration \( \varphi_0 < \varphi_c \), that is, the propagation velocity of the suspension-supernatant interface, is given by
\[ \sigma(0, \varphi_0) = \frac{f_{bk}(\varphi_0)}{\varphi_0}. \]  (20)

In addition to (19), the discontinuities must satisfy an admissibility criterion or entropy condition in order to ensure that the solution is physically relevant (Bustos et al., 1999). The numerical method of solution should

\[ L, \varphi_L, z_c / L, F, \varphi_F, \varphi_D, \varphi_c, \rho_s, \rho_l, u_s, c, \alpha_1, \alpha_2 \]

\[ \text{Determine } G(\varphi, \varphi_D) = \frac{1}{\rho_s f_{bk}(\varphi, \varphi_D)} \left( \frac{\varphi}{\varphi_D} - 1 \right) \]

\[ \text{Determine } UA = G(\varphi, \varphi_D), \quad \varphi_L \leq \varphi \leq \varphi_0 \]

\[ \text{Calculate } S = UA \times F \]

\[ \text{Calculate the concentration profile from } \frac{dz}{d\varphi} = -\frac{\Delta \rho \varphi g}{\sigma_c(\varphi)f_{bk}(\varphi)} \left( q(\varphi - \varphi_D) + f_{bk}(\varphi) \right) \]

\[ \text{Iterate changing } S \text{ to meet } z_c / L \approx 0.20 \]

\[ \text{Determine } UA, S \text{ and } \varphi_L \]

\[ L, T, \varphi_0, \varphi_c, \rho_s, \rho_l, u_s, u_{max}, c, \alpha_1, \alpha_2 \text{ or } \sigma_0 \text{ and } n \]

\[ \text{Determine } \Delta t \text{ from the CFL condition} \]

\[ \text{Solve the PDE } \frac{\partial \varphi}{\partial t} + \frac{\partial f_{bk}(\varphi)}{\partial z} = -\frac{\partial}{\partial z} \left( \frac{f_{bk}(\varphi) \sigma_c(\varphi)}{\Delta \rho \varphi g} \frac{d\varphi}{dz} \right) \]

Using the model functions
\[ f_{bk}(\varphi) = u_s \varphi \left( 1 - \varphi / \varphi_{max} \right) \]
\[ \sigma_c(\varphi) = \alpha_1 \exp(\alpha_2 \varphi) \text{ or } \sigma_c(\varphi) = \sigma_0 \left( \varphi / \varphi_c \right)^n - 1 \]
by the generalized upwind method with extrapolation

Fig. 6. Algorithm for the design of a continuous thickener.

Fig. 7. Algorithm for the simulation of batch sedimentation according to Garrido et al. (2003).
have a built-in property to reproduce appropriately discontinuities of the entropy solutions, especially the suspension–sediment interface, where the equation changes type, without the necessity to track them explicitly, that is, the scheme should possess the so called shock-capturing property. Another obvious requirement is that the scheme should converge to the correct (entropy) solution of the given problem. This clearly rules out classical schemes based on naive finite differencing for strictly parabolic equations, which otherwise work well for smooth solutions (Evje and Karlsen, 2000).

The scheme integrated into the software is a finite difference scheme having all these properties and can be referred to as generalized upwind scheme with extrapolation. Details of the scheme may be found in Bürger and Karlsen (2001). Fig. 7 shows the algorithm and Figs. 8 and 9 show the result of the simulation according to Garrido (2000).

5. Example of the design of a continuous thickener

Consider the design of an industrial thickener for a feed rate of 178 tons per hour, with an underflow concentration of 57.3% of solid by weight of a copper tailing with the following properties:

- Solid density \( \rho_s = 2500 \text{ kg/m}^3 \)
- Liquid density \( \rho_l = 1000 \text{ kg/m}^3 \)
- Feed concentration \( w_F = 35\% \) by weight
- Critical concentration \( w_c = 42.7\% \) by weight
- Setting parameters \( u_\infty = -0.000605 \text{ m/s}, \) \( c = 12.59, \phi_{\text{max}} = 1 \)
- Compression parameters \( \varphi_1 = 5.35 \text{ Pa}, \varphi_2 = 17.9 \)

The thickener height should be 6 m and the sediment height should not be higher that 20% of the total height. Choose \( \varphi_L = 0.01 \) (Fig. 10).

The maximum unit area is \( UA = 0.733 \text{ m}^2/\text{tpd} \) at 0.012 volume fraction of solids. With this result, run the optimization module adding the compression parameter information and the desired thickener height. The
simulator iterates until the desired height of sediment is obtained. The result is the thickener diameter \( D = 52.789 \) (Fig. 11).

Set the thickener diameter to the next integer, \( D = 53 \) m, and simulate again (Fig. 12).

The results of the design procedure are a unit area of \( UA = 0.516 \text{ m}^2/\text{tpd} \), a thickener diameter of \( D = 53 \) m and a conjugate concentration of \( \phi_L = 0.0176 \). A schematic version of the thickener with the mass balance and the concentration profile is shown in the next figures (Figs. 13 and 14).

6. Discussion and conclusions

In our previous paper, (Garrido et al., 2003) we presented an algorithm for the simulation of batch and continuous thickening. In this work, we extend that work to the design of industrial thickeners.

The first part of the design procedure makes use of the solution of the phenomenological model at steady state. The result is an equation similar to that of Coe and Clevenger (1916) that seeks for the maximum unit area obtained from the experimental data. Coe and
Clevenger’s design procedure is therefore supported by the presently accepted phenomenological theory. In our method, the whole function $G(\varphi, \varphi_D)$ is plotted, from which its maximum between the conjugate concentration $\varphi_L$ and the underflow concentration $\varphi_D$ may be obtained.

The problem to calculate the maximum of $G(\varphi, \varphi_D)$ is that the conjugate concentration $\varphi_L$ is unknown until the value of $F$ is known, and to calculate this term ($F = -F/\rho, S$) we need $S$, which is the result we are seeking for. Therefore, the problem is undetermined and, to solve it, a value of $\varphi_L$ must be guessed (for example $\varphi_L \approx 0.01$). On the other hand, a choice of $S$ leads to the calculation of $F$ and $q$ and, therefore, the calculation of the concentration profile in the thickener with Eq. (14). In this way, a value of the sediment height $z_c$ is obtained.

The next steps of the design are the choice of a determined value for $z_c$ (for example 20% of the height $L$ of the thickener) and iterate changing the value of $S$ to obtain a corrected unit area.

Finally, the thickener diameter obtained is approximated to the next integer and the simulation is repeated.
to obtain the conjugate concentration and the thickener area.

This paper reflects the present state of development of the software. The next step will be the addition of the process dynamics and control strategies.

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